Math 72

2.4 Equations of Lines and Linear Models

Objectives

2.4-1st:

1) Calculate the slope of a vertical line or horizontal line.

2) Visualize a line with positive slope, negative slope, zero slope, or undefined slope.

3) Write the equation of a vertical line or a horizontal line

4) Use the point-slope formula to write the equation of a line that is neither vertical nor horizontal.

5) Find the x-intercept and y-intercept of a line.

2.4-2nd

1) Use the slopes and y-intercepts of two lines to determine if they are
   a) parallel lines
   b) the same line
   c) perpendicular lines
   d) neither parallel nor perpendicular

2) Write a linear equation given an application problem.

3) Write the equation of a line parallel to a given line.

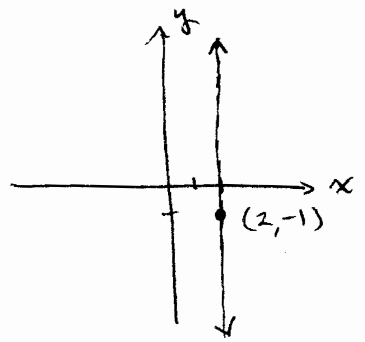
4) Write the equation of a line perpendicular to a given line.

① a) Graph a vertical line through the point $(2,-1)$

b) Find its slope.

c) Find its equation.

a)



step 1: plot the point $(2,-1)$

step 2: draw a vertical line through it.

b) slope formula $m = \frac{y_2 - y_1}{x_2 - x_1}$ requires a second pair.

choose any ordered pair on the vertical line

ex: $(2,0)$ or $(2,3)$ or $(2,-5)$

I choose $(2,0)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-(-1)}{2-2} = \frac{1}{0} = \boxed{\text{undefined}}$$

all vertical lines have undefined slope! * memorize

c) All points on a vertical line have the same x-coordinate.

This line $\boxed{x=2}$.

Equations of vertical lines are always $x = x\text{-coord}$ * memorize

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Question: Does the $y=mx+b$ method work for vertical lines?

$m =$ undefined is not a number

y-variable should not be in the answer.

No. The $y=mx+b$ method does not work.

vertical lines are a special case! watch for:

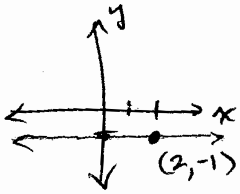
- "vertical"
- two points with same x-coordinate
- $\Delta x = 0$
- "undefined slope"
- parallel to another vertical line
- perpendicular to a horizontal line.

2.4-1st ② a) Graph a horizontal line through the point $(2,-1)$

b) Find its slope

c) Find its equation.

a)



b) choose a second ordered pair on the line

ex $(0,-1)$ $(3,-1)$ $(-2,-1)$

I choose $(0,-1)$

$$m = \frac{y_2-y_1}{x_2-x_1} = \frac{-1-(-1)}{2-0} = \frac{0}{2} = \boxed{0}$$

all horizontal lines have zero slope! * memorize

c) All points on a horizontal line have the same y-coord.

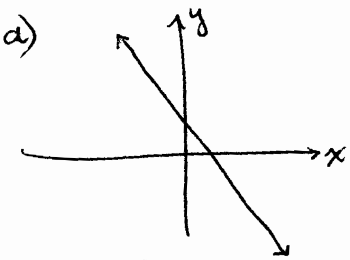
This line is $\boxed{y=-1}$ or as a linear function $\boxed{f(x)=-1}$.

Equations of horizontal lines are always y = y-coordinate

* memorize

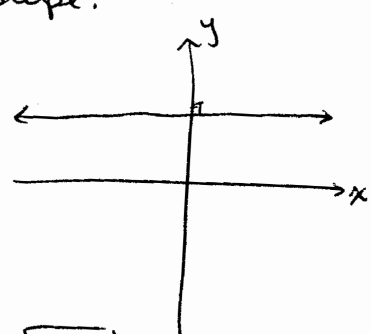
2.4-1st ③ For each graph, identify if the slope is positive, negative, zero or undefined without calculating the slope.

a)



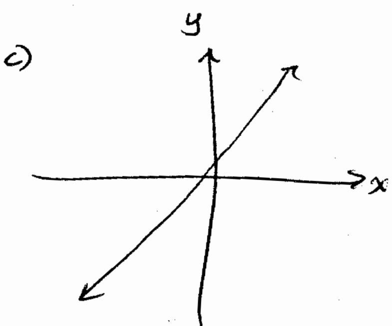
$\boxed{\text{negative}}$

b)



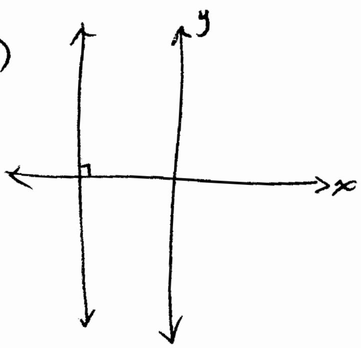
$\boxed{\text{zero}}$

c)



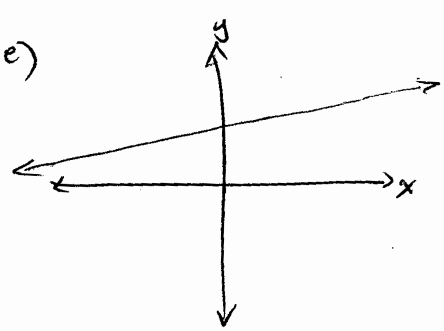
$\boxed{\text{positive}}$

d)



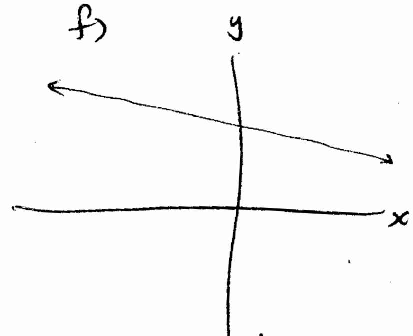
$\boxed{\text{undefined}}$

e)



$\boxed{\text{positive}}$

f)



$\boxed{\text{negative}}$

(4) Use the point-slope formula to find the equation of a line through $(5, 2.5)$ and $(-.5, 3)$.

Step 1: Find the slope of the line.
We did this in 2.3!
$m = \frac{-1}{11}$

Point-Slope Formula $y - y_1 = m(x - x_1)$

(x_1, y_1) is a point on the line

$m =$ slope

x and y are the variables that remain in the final answer

* Memorize

Method 1: use $(5, 2.5)$ as (x_1, y_1)

$y - y_1 = m(x - x_1)$

$y - 2.5 = \frac{-1}{11}(x - 5)$ substitute $m = -\frac{1}{11}$, $x_1 = 5$, $x_2 = 2.5$

$y - 2.5 = \frac{-1}{11}x + \frac{5}{11}$

$y = \frac{-1}{11}x + \frac{5}{11} + 2.5$

$\boxed{y = \frac{-1}{11}x + \frac{65}{22}}$

Method 2: use $(-0.5, 3)$ as (x_1, y_1)

$y - y_1 = m(x - x_1)$

$y - 3 = \frac{-1}{11}(x - (-0.5))$

$y - 3 = \frac{-1}{11}(x + 0.5)$

$y - 3 = \frac{-1}{11}x - \frac{1}{22}$

$y = \frac{-1}{11}x - \frac{1}{22} + 3$

$\boxed{y = -\frac{1}{11}x + \frac{65}{22}}$

Method 3: Use slope-intercept form and $(5, 2.5)$ as (x, y)

$y = mx + b$ slope-intercept form

$2.5 = -\frac{1}{11}(5) + b$ substitute $y = 2.5$, $x = 5$, $m = -\frac{1}{11}$

$2.5 = \frac{-5}{11} + b$ solve for b.

$2.5 + \frac{5}{11} = b$

$b = 2.9\overline{54} = \frac{65}{22}$

$$\boxed{y = \frac{-1}{11}x + \frac{65}{22}}$$

subst. values of m and b into $y = mx + b$

Method 4: Use slope-intercept form and $(-.5, 3)$ as (x, y).

$y = mx + b$

Substitute $y = 3$, $x = -.5$, $m = \frac{-1}{11}$

$3 = \frac{-1}{11}(-.5) + b$

Solve for b

$3 = \frac{.5}{11} + b$

$3 = \frac{1}{22} + b$

$3 - \frac{1}{22} = b$

$\frac{65}{22} = b$

$$\boxed{y = \frac{-1}{11}x + \frac{65}{22}}$$

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(5) Find the x-intercept and y-intercept for each line. Write answers as ordered pairs. Sketch graph.

a) $2x - 4y = 11$

x int: set $y = 0$

$2x - 4(0) = 11$

$x = \frac{11}{2}$ $\boxed{\left(\frac{11}{2}, 0\right) \text{ x int}}$

$\frac{11}{2} = 5.5$

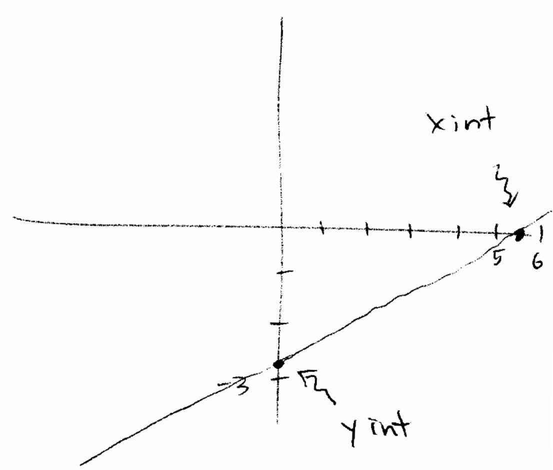
y-int: set $x = 0$

$2(0) - 4y = 11$

$-4y = 11$

$y = \frac{-11}{4}$ $\boxed{\left(0, \frac{-11}{4}\right) \text{ yint}}$

$\frac{-11}{4} = -2.75$



b) $y = 3x + 5$

x int: set $y = 0$

$0 = 3x + 5$

$-5 = 3x$

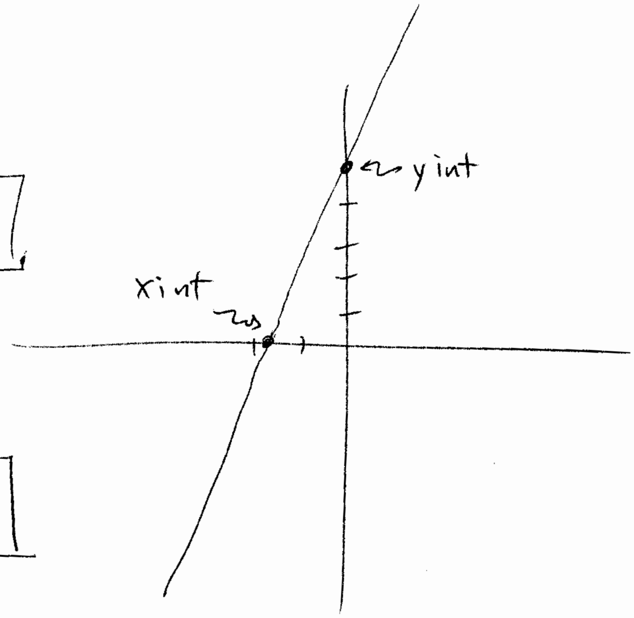
$\frac{-5}{3} = x$ $\boxed{\left(-\frac{5}{3}, 0\right) \text{ x int}}$

$-\frac{5}{3} = -1\frac{2}{3}$

y int set $x = 0$

$y = 3(0) + 5$

$y = 5$ $\boxed{(0, 5) \text{ y-int}}$



c) $y = -3$

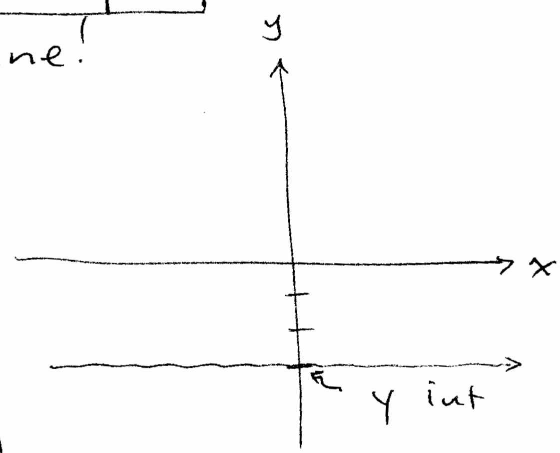
x int: set $y = 0$

$0 \neq -3$ false! This equation is a contradiction and has no solution.

There is no x-intercept.

WAKE UP! It's a horizontal line!

$\boxed{\text{y int } (0, -3)}$

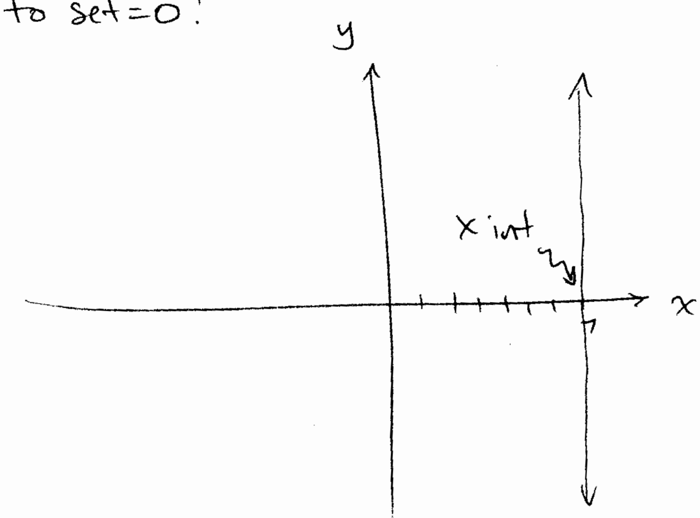


d) $x = 7$

$\boxed{\text{x int } (7, 0)}$ no y to set = 0!

$\boxed{\text{y int none}}$

It's vertical!



Mixed Practice

Write the equation of the line having the given characteristics. Write final answers in the form $y = mx + b$ if possible.

Sketch the graph of the line.

Find the x-intercept and y-intercept and write each as an ordered pair.

Label the x-intercept and y-intercept on your graph.

a) Passes through $(4.5, 1)$ and $(-4.5, -1)$

b) Passes through $(4.5, 1)$ and $(-4.5, 1)$

c) Passes through $(-4.5, 1)$ and $(-4.5, -1)$

a) Passes through $(4.5, 1)$ and $(-4.5, -1)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-(-1)}{4.5-(-4.5)} = \frac{2}{9}$$

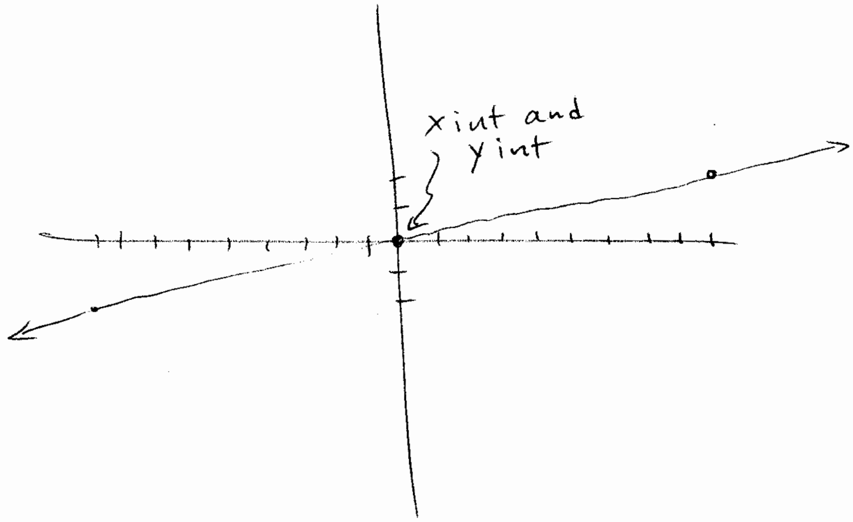
$$y - 1 = \frac{2}{9}(x - 4.5)$$

$$y - 1 = \frac{2}{9}x - 1$$

$$\boxed{y = \frac{2}{9}x}$$

$y = \frac{2}{9}x + 0$

$\boxed{\begin{array}{l} y\text{ int } (0,0) \\ x\text{ int } (0,0) \end{array}}$

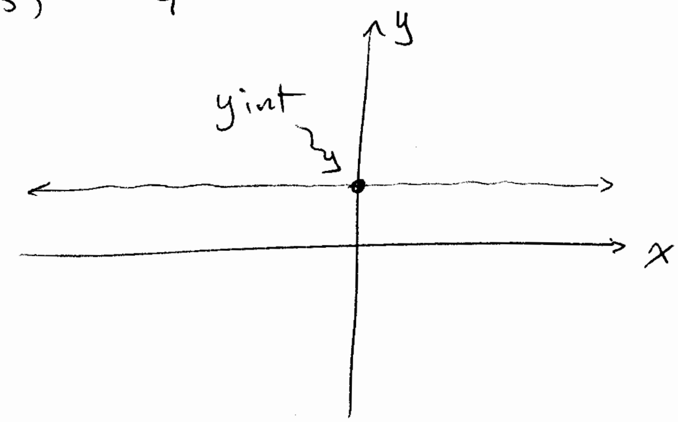


b) Passes through $(4.5, 1)$ and $(-4.5, 1)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-1}{4.5-(-4.5)} = \frac{0}{9} = 0 \quad \text{horizontal!}$$

$\boxed{y = 1}$

$\boxed{\begin{array}{l} y\text{ int } (0,1) \\ \text{no } x\text{ int} \end{array}}$

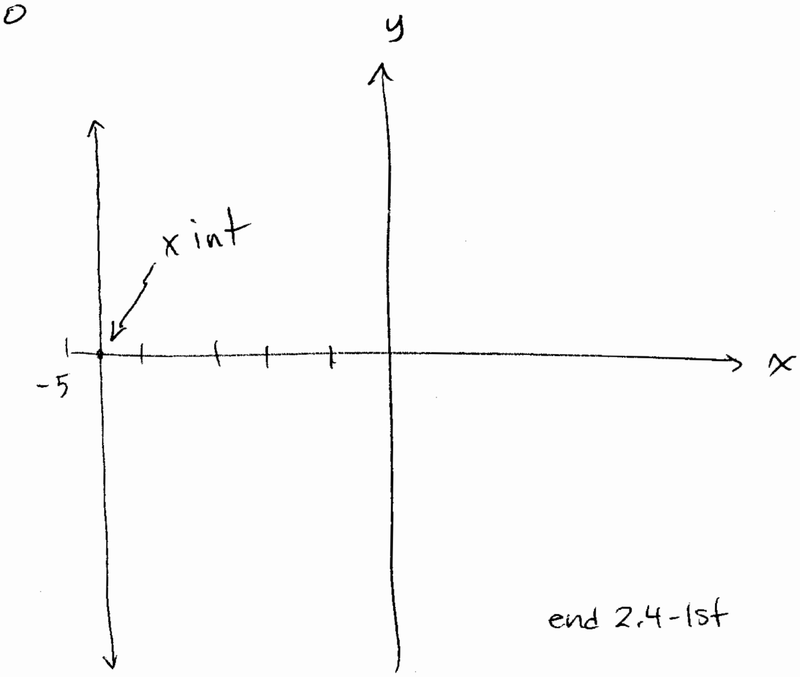


c) Passes through $(-4.5, 1)$ and $(-4.5, -1)$

$$m = \frac{1-(-1)}{-4.5-(-4.5)} = \frac{2}{0} = \text{undefined} \quad \text{vertical!}$$

$\boxed{x = -4.5}$

$\boxed{\begin{array}{l} \text{no } y\text{ int} \\ x\text{ int } (-4.5, 0) \end{array}}$



end 2.4-1st

2.4-2nd

1. Graph the lines using slope and y-intercept, on the same axes. Identify which are the same line, parallel lines, perpendicular lines, or neither parallel nor perpendicular

a) $y = \frac{3}{2}x + 1$

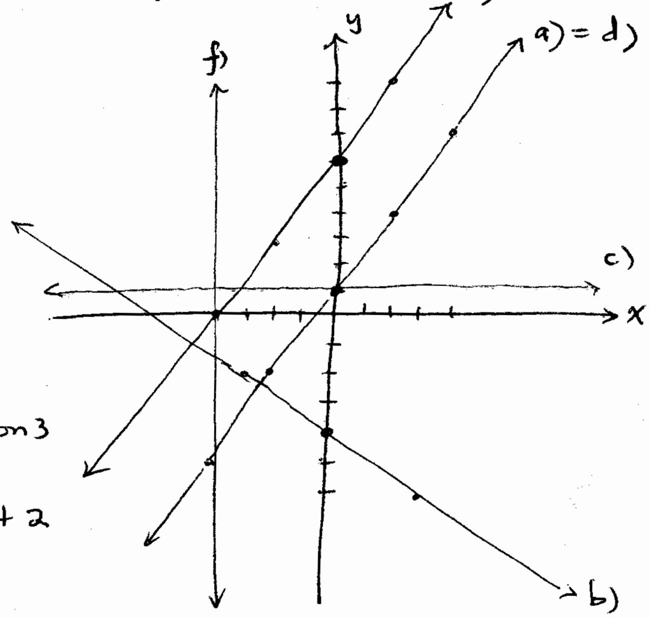
b) $y = -\frac{2}{3}x - 4$

c) $y = 1$

d) $3x - 2y = -2$

e) $f(x) = \frac{3}{2}x + 6$

f) $x = -4$



a) $y = \frac{3}{2}x + 1$

slope $m = \frac{3}{2}$ $\frac{\nearrow \text{up } 3}{\rightarrow \text{right } 2}$ or $\frac{3}{2} = \frac{-3}{-2}$ $\frac{\nearrow \text{down } 3}{\rightarrow \text{left } 2}$

y intercept 1 $\Rightarrow$ $(0,1)$

b) $y = \frac{-2}{3}x - 4$

slope $m = \frac{-2}{3}$ $\frac{\nearrow \text{down } 2}{\rightarrow \text{right } 3}$ or $\frac{2}{-3}$ $\frac{\nearrow \text{up } 2}{\rightarrow \text{left } 3}$

y intercept -4 $\Rightarrow$ $(0,-4)$

c) $y = 1$

$y = 0x + 1$

slope $m = 0$, horizontal

y int 1 $\Rightarrow$ $(0,1)$

d) $3x - 2y = -2$

Solve for y: subtract 3x both sides

$-2y = -3x - 2$

divide by -2 both sides.

$y = \frac{-3}{-2}x - \frac{2}{-2}$

$y = \frac{3}{2}x + 1$ same as a)

e) $f(x) = \frac{3}{2}x + 6$

$m = 3/2$

y int $(0,6)$

f) $x = -4$

vertical

through $(-4, \text{any})$

	a	b	c	d	e	f
a	S	⊥	N	S	∥	N
b	⊥	S	N	⊥	⊥	N
c	N	N	S	N	N	⊥
d	S	⊥	N	S	∥	N
e	∥	⊥	N	∥	S	N
f	N	N	⊥	N	N	S

a) and d) are the same line [same slope, same y-intercept]

a) is parallel to e) and d) is parallel to e) [same slope, diff y-int]

a) is perpendicular to b), d) is perpendicular to b) e) is ⊥ to b)

c) is perpendicular to f) [opposite & reciprocal slopes] OR [vertical ⊥ horizontal]

2.4-2nd (2) A 20,000-gallon swimming pool is filled at a constant rate. Over a 5-hour period, the water in the pool increases from $\frac{1}{4}$ full to $\frac{5}{8}$ full. a) At what rate is water entering the pool?

b) Write a linear equation describing the quantity of water in the pool after x hours.

c) Interpret the slope.

$\frac{1}{4}$ full $\Rightarrow$ $\frac{1}{4}$ of 20000 $= \frac{1}{4}(20000) = 5000$ gallons at start $x=0$h

$\frac{5}{8}$ full $\Rightarrow$ $\frac{5}{8}$ of 20000 $= \frac{5}{8}(20000) = 12500$ gallons at $x=5$ hours.

a) rate = slope $= \frac{y_2-y_1}{x_2-x_1} = \frac{12500-5000}{5-0} = \boxed{1500\ \text{gal}/\text{hr}}$

Method 1:

b) $y-y_1 = m(x-x_1)$

$y-12500 = 1500(x-5)$

$y-12500 = 1500x-7500$

$\boxed{y = 1500x+5000}$

or

Method 2:

$y = mx+b$

$\boxed{y = 1500x+5000}$

c) $m = 1500\ \frac{\text{gal}}{\text{hr}}$

The amount of water in the pool increases by 5000 gallons per hour.

2.4-2nd (3) Write the equation of a line passing through $(3,-9)$

a) parallel to $y = -\frac{1}{3}x-4$

b) perpendicular to $y = -\frac{1}{3}x-4$

c) Graph and label all three lines.

a) $m = -\frac{1}{3}$ $\Rightarrow$ parallel means same slope

$y-y_1 = m(x-x_1)$

$y-(-9) = -\frac{1}{3}(x-3)$

$y+9 = -\frac{1}{3}x+1$

$\boxed{y = -\frac{1}{3}x-8}$

2.4 #nd b) perpendicular means slope is opposite and reciprocal

$m = -\frac{1}{3} \implies m = +\frac{3}{1} = 3$

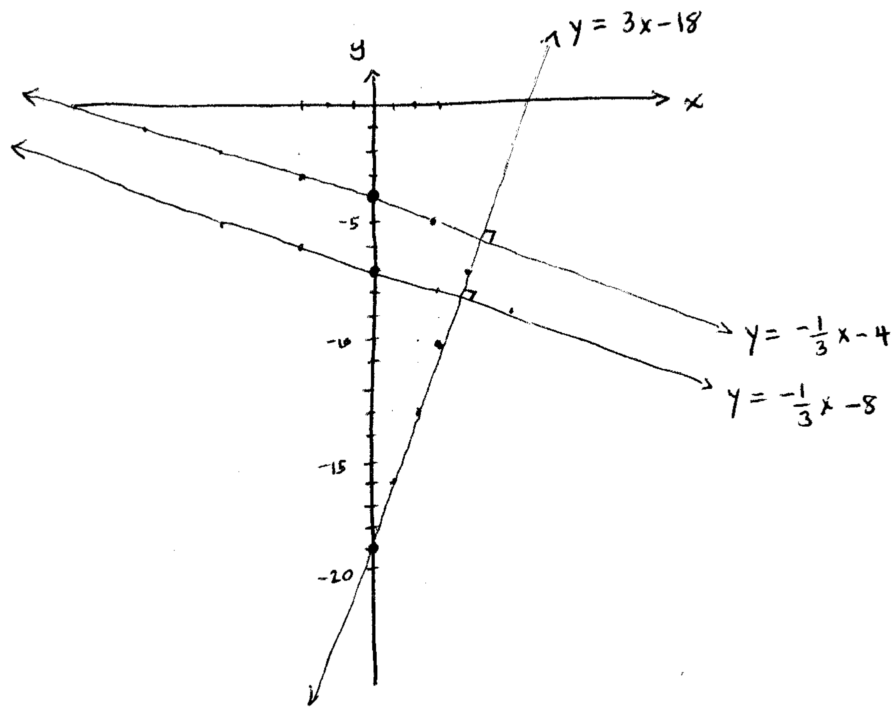
$y - y_1 = m(x - x_1)$

$y - (-9) = 3(x - 3)$

$y + 9 = 3x - 9$

$\boxed{y = 3x - 18}$

c)



M72 2.3 & 2.4 Write the Equation of a Line as a Linear Function

CASE 1: Line is vertical. Write equation $x = x_{coordinate}$
How to know if a line is vertical?

- It says "vertical".
- It says "slope undefined".
- It is parallel to a line with undefined slope.
- It is parallel to a vertical line $x = x_{coordinate}$.
- It is perpendicular to a horizontal line, $f(x) = y_{coordinate}$ or $y = y_{coordinate}$
- It is perpendicular to a line with slope = 0.
- It is parallel to the y-axis.
- It is perpendicular to the x-axis.

IMPORTANT: Vertical lines are not functions!
IMPORTANT: Do NOT use $f(x) = mx + b$ or $y - y_1 = m(x - x_1)$!

Case 2: Line is horizontal. Write equation. $f(x) = y_{coordinate}$
How to know if a line is horizontal?

- It says "horizontal".
- It says "slope 0".
- It is parallel to a line with zero slope.
- It is parallel to a horizontal line $f(x) = y_{coordinate}$.
- It is perpendicular to a vertical line, $x = x_{coordinate}$.
- It is perpendicular to a line with undefined slope.
- It is parallel to the x-axis.
- It is perpendicular to the y-axis.

Case 3: Given a non-zero slope and a point (x_1, y_1):
If the point is the y-intercept $(x_1, y_1) = (0, b)$, substitute into $f(x) = mx + b$.
If the point is not the y-intercept,

Option 1: Use the point-slope formula $y - y_1 = m(x - x_1)$: substitute the slope for m and the point (x_1, y_1) [or (x_2, y_2)] for (x_1, y_1) into the formula, simplify and isolate y, and replace y by $f(x)$.

Option 2: Use the slope-intercept form $f(x) = mx + b$: replace f(x) by y_1, x by x_1, [or f(x) by y_2, x by x_2] and the slope by m, then solve for b. Rewrite the final answer by substituting m and b into $f(x) = mx + b$.

Case 4: Given two points (x_1, y_1) and (x_2, y_2):
Find the slope using the slope formula: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$. Proceed as in Case 3.

Case 5: Given "parallel to _____" and a point (x_1, y_1):
Find the slope of the given line by rearranging it to slope-intercept form $y = mx + b$.
Use that same slope.
Proceed as in Case 3.

Case 6: Given "perpendicular to _____" and a point (x_1, y_1):
Find the slope of the given line by rearranging it to slope-intercept form $y = mx + b$.
Take the opposite and reciprocal of that slope to get the new slope.
Proceed as in Case 3.

MATH 45 Summary of Techniques for Graphing A Line

Method 1: Plot x-intercept, Plot y-intercept

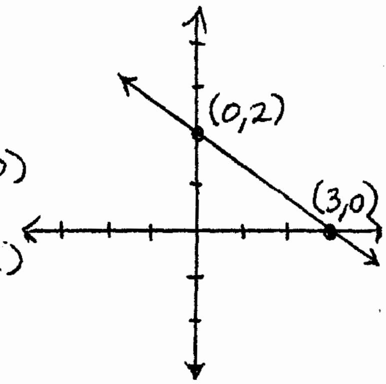
Use if both the x-intercept and y-intercept are integers.
To find x-intercept, set $y=0$, solve for x.
To find y-intercept, set $x=0$, solve for y.

Example: $2x+3y=6$

x-int: $2x=6$
$x=3 \rightarrow (3,0)$

y-int: $3y=6$
$y=2 \rightarrow (0,2)$

Use this method if the constant (6) can be evenly divided by either coefficient (2 or 3).



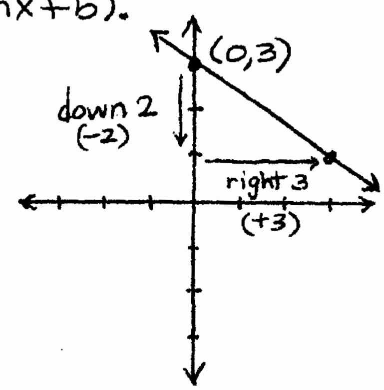
Method 2: Plot y-intercept, Use slope

Use if the y-intercept is an integer, but the x-intercept is not.
Write the equation in slope-intercept form ($y=mx+b$).

Example:

$$2x+3y=9$$
$$-2x \qquad -2x$$
$$\frac{3y}{3}=\frac{-2x}{3}+\frac{9}{3}$$
$$y=\frac{-2}{3}x+3 \longrightarrow$$

y int $=(0,3)$
slope $=\frac{-2}{3}$



Method 3: Plot x-intercept, Use slope

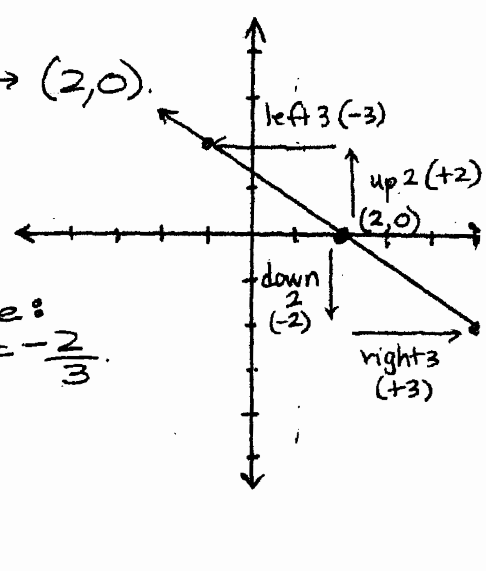
Use if the x-intercept is an integer, but the y-intercept is not.
To find x-intercept, set $y=0$, solve for x.
Write equation in slope-intercept form ($y=mx+b$).

Example: $2x+3y=4$

x-int: $2x=4$
$x=2 \rightarrow (2,0)$.

$$2x+3y=4$$
$$-2x \qquad -2x$$
$$\frac{3y}{3}=\frac{-2x}{3}+\frac{4}{3}$$
$$y=-\frac{2}{3}x+\frac{4}{3} \longrightarrow$$

slope:
$m=-\frac{2}{3}$.



Method 4: Plot any point, use slope

Use if neither the x-intercept nor the y-intercept is an integer.
Write the equation in slope-intercept form ($y=mx+b$).
Make a chart:

x	y
1	
2	
3	

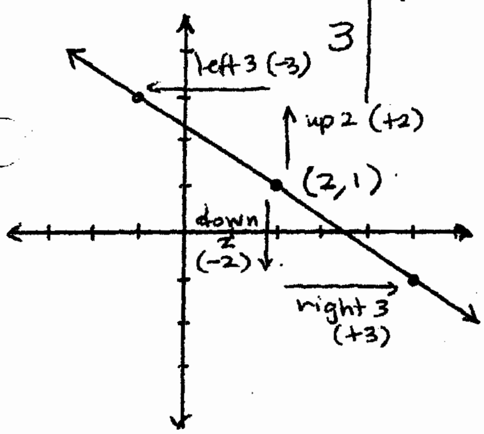
Stop work on the chart when you find one point (x,y) where both coordinates are integers.

Example: $2x+3y=7$

$$\begin{aligned} 2x + 3y &= 7 \\ -2x \qquad &\quad -2x \\ \frac{3y}{3} &= \frac{-2x}{3} + \frac{7}{3} \\ y &= -\frac{2}{3}x + \frac{7}{3} \end{aligned}$$

$\longrightarrow$ slope $m = -\frac{2}{3}$.

x	y
1	$5/3$
2	1
3	



$x=1$: $2(1)+3y=7$

$$\begin{aligned} -2 \qquad &\quad -2 \\ \frac{3y}{3} &= \frac{5}{3} \\ y &= \frac{5}{3} \end{aligned}$$

keep going.

$x=2$: $2(2)+3y=7$

$$\begin{aligned} -4 \qquad &\quad -4 \\ \frac{3y}{3} &= \frac{3}{3} \\ y &= 1. \end{aligned}$$

stop.

Method 5: Plot any two points.

Use if neither the x-intercept nor the y-intercept is an integer.
Make a chart

x	y
1	
2	
3	

Stop work on the chart when you find two points (x,y) where both coordinates are integers.

Example: $2x+3y=7$

x	y	
1	$5/3$	
2	1	$\rightarrow (2,1)$
3	$1/3$	
4	$-1/3$	
5	-1	$\rightarrow (5,-1)$